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REPORT NO. 57

TITLE OF REPORT: Determination of k_c, k_ϕ, n-Values by Means of Circular Footings, Modified Procedure

by

B. Hanamoto & Z. Janosi

This is a working paper presenting the considered results of a study by the staff of the Land Locomotion Laboratory, Research Division, Research and Engineering Directorate, Ordnance Tank-Automotive Command.

November 1959

DEPARTMENT OF THE ARMY
ORDNANCE TANK-AUTOMOTIVE COMMAND
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The findings and analysis are subject to revision, or may be required by new facts or modification of basic assumptions. Comments and criticisms should be addressed to:

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Center Line, Michigan
ATTENTION: ORDMC-RRL
Land Locomotion Laboratory

DEPARTMENT OF THE ARMY
ORDNANCE TANK-AUTOMOTIVE COMMAND
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Mr. Server Tasdemiroglu, Mr. James Landoll and Lt. Gordon Bassett helped with the tests and evaluation.

ABSTRACT

It has been shown by means of theoretical and experimental methods that the size effect of a circular footing penetrating vertically into soil has to be considered by applying to equation $p = (\frac{k_c}{b} + k_\phi)z^n$ the radius of the plate instead of the diameter. Slight discrepancies formerly noted between soil values established by means of rectangular and circular plates can be thus eliminated.

Determination of k_c, k_ϕ, n-Values by Means of Circular Footings, Modified Procedure

Problem

Since soil values k_c, k_ϕ and n obtained by means of circular and rectangular plates show some discrepancies, investigate the existing method of soil value evaluations and suggest necessary modifications in order to obtain agreements with practical limits.

Background

The Land Locomotion Laboratory introduced and successfully used a semi-empirical stress-strain relationship for vertical soil deformation (1).

$$p = (k_c/b + k_\phi)z^n$$

where

p is the vertical unit load (psi)

k_c is the "cohesive modulus" of soil deformation (lb/in^{n+1})

k_ϕ is the "frictional modulus" of soil deformation (lb/in^{n+2})

z is the vertical sinkage corresponding to p (in)

b is the width of the rectangular footing (in)

When obtaining soil values by means of a circular footing, the diameter (D) has been used in the denominator of k_c/b.

The method for the evaluation of soil value k_c, k_ϕ, and n has been described in detail in numerous papers published by this organization (2).

Essentially it consists of the following steps:

1. Obtain experimental p vs z curves by at least two plates of different dimensions say; b_1 and b_2 or D_1 and D_2.

Then:

$$p = \left(\frac{k_c}{b_1} + k_\phi\right)z^n \qquad p = \left(\frac{k_c}{b_2} + k_\phi\right)z^n$$

2. Plot the empirical curves on logrithmic paper.

$$\ln p = \ln \left(\frac{k_c}{b_1} + k_\phi\right) + n \ln z$$

$$\ln p = \ln \left(\frac{k_c}{b_2} + k_\phi\right) + n \ln z$$

which yield two parallel straight lines.

3. Determine the slope of the two lines, (this slope defines the value n) and ordinates a_1 and a_2 (at $z = 1$). Accordingly:

$$a_1 = \frac{k_c}{b_1} + k_\phi \qquad (2)$$

$$a_2 = \frac{k_c}{b_2} + k_\phi \qquad (3)$$

As mentioned before, the use of D for b when using circular plates does not produce quite the same soil values as obtained by rectangular plate tests with large aspect ratio. However, since k_c / b is considerably smaller than k_ϕ in most cases, these differences have not been significant in the early stages of the application of the discussed soil value system, and remained unnoticed.

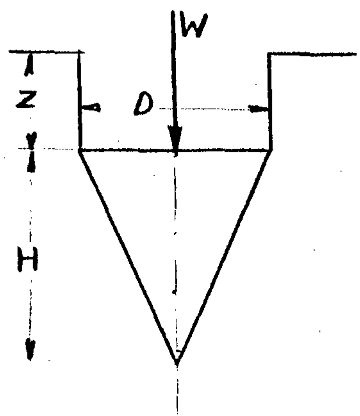
In the course of another research project (3), however, it became evident that the situation could be improved by using R instead of D and that such a procedure would be in a better agreement with theoretical concept of the discussed values. Theoretical background of this conclusion is outlined below.

Discussion of the Theory

As shown in an earlier report (3) the force needed to penetrate a cone into the soil can be expressed by the following equation:

$$W = \frac{\pi D}{2H}\left[\frac{k_c}{n+1}\left\{(z+H)^{n+1} - z^{n+1}\right\} + \frac{k_\phi D}{H}\left\{\frac{(z+H)^{n+2}}{(n+1)(n+2)} + \frac{z^{n+2}}{n+2} - \frac{z^{n+1}(z+H)}{n+1}\right\}\right] \qquad \text{------------ } 4$$

Where D (in) is the diameter of the cone-base; H (in) is height (in) of the cone; z is the sinkage (in) measured between the ground surface and the cone base (Figure 1). When $H \rightarrow 0$ Equation 4 furnishes the force needed to penetrate a circular plate into the ground. Since one would encounter the indeterminate form of $\frac{0}{0}$ during the mathematical procedure L' Hospitalds rule has to be applied.



Accordingly

$$\lim_{H \rightarrow 0} \frac{f(H)}{\phi(H)} = \frac{f'(H_o)}{\phi'(H_o)}$$

and if

$$\frac{f'(H_o)}{\phi'(H_o)} \quad \text{is still indeterminate}$$

then

$$\lim_{H \rightarrow 0} \frac{f(H)}{\phi(H)} = \frac{f''(H_o)}{\phi''(H_o)}$$

Let us divide Equation 4 into two parts

$$W_1 = \frac{\pi D k_c}{2(n+1)} \left[\frac{(z+H)^{n+1} - z^{n+1}}{H} \right]$$

and

$$W_2 = \frac{\pi D^2}{2} k_\phi \left[\frac{(z+H)^{n+2}}{(n+1)(n+2)H^2} + \frac{z^{n+2}}{(n+2)H^2} - \frac{z^{n+1}(z+H)}{(n+1)H^2} \right]$$

The numerators of W_1 and W_2 are denoted $f_1(H)$ and $f_2(H)$ respectively, whereas the denominators are called $\phi_1(H)$ and $\phi_2(H)$.

So:

$$f_1(H) = \pi D k_c \left[(z+H)^{n+1} - z^{n+1} \right]$$

$$\phi_1(H) = 2(n+1)H$$

$$f_2(H) = \pi D^2 k_\phi \left[(z+H)^{n+2} + (n+1) z^{n+2} - (n+2) z^{n+1} (z+H) \right]$$

$$\phi_2(H) = 2(n+1)(n+2)H^2$$

The first derivatives are:

$$f_1'(H) = \pi D k_c (n+1)(z+H)^n$$

and for $H = 0$,

$$f_1'(H_o) = \pi D k_c (n+1) z^n$$

$$\phi_1'(H) = 2(n+1) = \phi_1'(H_o)$$

So

$$W_{1o} = \frac{f_1'(H_o)}{\phi_1'(H_o)} = \frac{\pi D k_c z^n}{2}$$

On the other hand:

$$f_2''(H) = \pi D^2 k_\phi (n+2)(n+1)(z+H)^n$$

$$\phi_2''(H) = 4(n+1)(n+2)$$

and

$$W_{2o} = \lim_{H \to 0} \frac{f''_2(H)}{\phi''_2(H)} = \frac{\pi D^2 k_\phi}{4} z^n$$

The total force, when $H \to 0$ equals:

$$W_o = W_{1o} + W_{2o} = \frac{\pi D k_c}{2} z^n + \frac{\pi D^2 k_\phi}{4} z^n$$

or

$$W_o = \frac{\pi D^2}{4} \left(\frac{2k_c}{D} + k_\phi\right) z^n \qquad \text{------ } 5$$

The unit load or pressure is then:

$$p = \frac{W_o}{\frac{\pi D^2}{4}} = \left(\frac{2k_c}{D} + k_\phi\right) z^n$$

Or if $R = \frac{D}{2}$

$$p = \left(\frac{k_c}{R} + k_\phi\right) z^n \qquad \text{------ } 6$$

Thus if Equation 4 is accepted as valid for a cone shaped footing then the use of R (radius) is verified by theoretical considerations.

Tests

Experimental verification of using the radius of a circular plate for b in Equation 1 was carried out by performing sinkage tests using both rectangular and circular plates.

Sinkage experiments in the laboratory were performed in an artificial soil mixture (4) of bentonite and ethylene glycol anti-freeze at four different liquid contents. The plate sizes for laboratory tests using a Bevameter Mark IV (2) were rectangular plates: 5 x 0.75; 5 x 1.5; 3 x 0.75; 3 x 1 and circular plates of diameters 0.8; 1.6; 2.0 and 3.0 (all dimensions in inches).

Tabulated below are the results of tests performed in the Laboratory.

Date	Wht % AF	Plate	k_ϕ	n	k_c / Rect	k_c / b= R	k_c / b= D
27 Aug 58	60	Circ	2.95	0.27		13.1	26.2
		Circ	23.1	0.36		13.0	26.0
		Rect	22.0	0.33	13.5		
		Rect	22.8	0.24	14.1		
28 Aug 58	60	Rect	26.0	0.16	12.3		
		Rect	29.6	0.29	9.6		
29 Aug 58	60	Circ	28.2	0.1		16.8	33.6
		Circ	28.2	0.17		14.4	28.8
		Rect	26.8	0.1	15.9		
2 Sep 58	65	Rect	14.0	0.14	16.8		
		Rect	11.6	0.18	22.8		
		Circ	15.0	0.16		20	20
		Circ	16.8	0.11	15.9		
3 Sep 58	65	Rect	20	0.26	12.0		
		Circ	19	0.1		18.4	36.8
		Rect	18	0.22	12.9		
10 Sep 58	67	Circ	8.6	0.17		3.3	6.6
		Rect	10.7	0.18	3.6		
		Rect	9.9	0.17	2.4		
		Rect	10.3	0.17	4.0		
		Rect	8.6	0.21	3.9		
		Rect	10.0	0.21	1.8		
		Rect	9.3	0.21	3.2		
		Circ	8.8	0.25		3.5	7.0
		Circ	7.4	0.25		5.0	10.0

Notice the reasonable agreement between the values in columns 6 and 7 and the disagreement in columns 6 and 8.

Aberdeen Proving Ground

Field tests at APG, Md., Churchville Test Area were conducted using the Mark V Bevameter (2). Even under field conditions the circular plate size factor or b= R is apparent. The plate sizes were rectangular plates 8x1, 8x1.5, 8x1.0, 6x0.75, 6x1, 6x1.5, and circular plates of diameters equal to 4.0, 5.0, 7.0 inches. The results are tabulated below:

Date	MC%*	Plate	k_ϕ	n	k_c Rect	k_c b= R	k_c b= D
3 Nov 58		Rect	1.2	0.49	0.66		
4 Nov 58	32	Circ	1.2	0.42		0.72	1.44
	31	Rect	1.3	0.32	0.8		
	33	Rect	1.7	0.33	0.9		
	32	Rect	1.4	0.44	0.66		
5 Nov 58	29	Rect	1.3	0.36	3.2		
6 Nov 58	29	Rect	2.7	0.63	1.6		
	28	Circ	1.8	0.4		4.8	9.6
7 Nov 58	25	Circ	3.0	0.6		1.12	2.24
	26	Rect	2.8	0.58	1.6		

*Moisture Content %

Discussion of Results

It is felt that the agreement between tested k_c values both by rectangular and circular plates when using b= R in Equations 2 and 3 leads to the conclusion that k_c is independent of the form of the footing used.

Recommendation

It is recommended that the vertical stress-strain relationship be used according to Equation 6 for circular plates.

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b. Research Report No. 2

c. Research Report No. 3

d. Research Report No. 4

e. Research Report No. 5

f. A Practical Outline of the Mechanics of Automotive Land Locomotion (Seminar notes presently out of print)

g. Interservice Vehicle Mobility Symposium, held at Stevens Institute of Technology, Hoboken, New Jersey, 18-20 April 1955

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